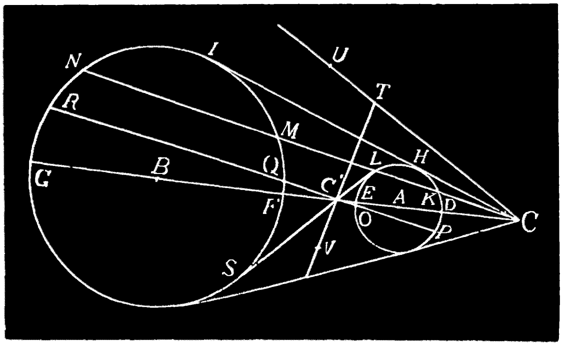
I. If $CKLMN$ be any secant line from C, then $CL\times CM=CK\times CN=CH\times CI$.

II. If $POC'QR$ be any line through C', secant to both circles, $C'O\times C'R=C'Q\times C'P=C'L\times C'S$.

III. If any circle be drawn to which both circles are either internally or externally tangent, the line through the points of tangency will pass through the external center of similitude, C.

IV. If any circle be drawn to which one of the given circles is internally tangent, and the other externally tangent, the line joining the points of tangency will pass through the internal center of similitude, C'.



Let T be the given point. On the line through CT take a point U such that $CT\times CU=CH\times CI$. This may be done by passing a circle through I, T, and H. It will cut CT in U.

The circle to which both circles are either internally or externally tangent and which passes through T, will also pass through U.

This follows from I. and III.

If through U and T we pass two circles tangent to either of the given circles, (Prob. 17.) they will be tangent to the other.

On the line through $C'T$ take a point V such that $C'T\times C'V=C'S\times C'L$. This may be done by passing a circle through T, L, and S. It will cut $C'T$ in V.

The circle to which one of the given circles is tangent internally and the other externally and which passes through T will also pass through V. This follows from II. and IV.

If through V and T we pass a circle to which one of the given circles is internally tangent, the other will touch it externally and *vice versa*.

Excellent solutions received from *Professors G. B. M. ZERR, P. H. PHILBRICK*, and *H. C. WHITAKER*.

15. Proposed by ISAAC L BEVERAGE, Monterey, Virginia.

A man starts from the center of a circular 10 acre field and walks due north a certain distance, then turns and walks south-west till he comes to the circumference, walking altogether 40 rods. How far did he walk before making the turn?

Solution by Professor J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

Let x be the first distance, y the second, and R the radius, then $x+y=40$....(1) and $x^2+y^2-2xy\cos 45^\circ=R^2$, or $x^2-xy\sqrt{2}+y^2=R^2=\dfrac{1600}{\pi}$....(2).

Squaring (1) and subtracting (2) from it, we have $xy=800\left(1-\frac{1}{\pi}\right)$ $(2-\sqrt{2})\ldots.(3)$. Now we at once get $y-x=40\sqrt{1-2\left(1-\frac{1}{\pi}\right)(2-\sqrt{2})}\ldots.(4)$.

Combining (1) and (2), we finally obtain

$$x=40\left[1-\sqrt{1-2\left(1-\frac{1}{\pi}\right)(2-\sqrt{2})}\right],\quad y=40\left[1+\sqrt{1-2\left(1-\frac{1}{\pi}\right)(2-\sqrt{2})}\right].$$

Also solved by *P. H. PHILBRICK*, *G. B. M. ZERR*, *H. M. CASH*, *P. S. BERG*, *CHARLES E. MYERS*, *J. A. CALDERHEAD*; *SETH PRATT*, and *H. C. WHITAKER*.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

16. Proposed by F. P. MATZ, M. Sc., Ph.D., Professor of Mathematics and Astronomy, in New Windsor College, New Windsor, Maryland.

Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with regard to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Solution by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland. and CHARLES E. MYERS, Canton, Ohio.

Let $\frac{2x}{1-x^2}=z$, then $\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}z$; but $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ by Trigonometry equals $\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}z$. Hence $\frac{dz}{dz}=1$, that is, since both expressions are identical, the first differential coefficent is$=1$.

Also solved by Professor *MATZ*, *SCHMITT*, and *ZERR*.

17. Proposed by H. W. DRAUGHON, Clinton, Louisiana.

To find the volume generated by revolving a circular segment whose base is a given chord, about any diameter as an axis.

Solution by the PROPOSER.

In the circle, center C, draw any diameter ECF, and also any chord